

## Chapter 3: Modern Portfolio Theory

### Exercises

1. You decide to invest your money in a stock portfolio consisting of 60% Cisco Systems and 40% Amazon.com. Using the data in Table 3.7 in the book you find that Cisco has an annual standard deviation of 0.363 and Amazon of 0.34. The correlation coefficient between the returns of both stocks is 0.34
  - (a) Calculate the variance and standard deviation of this portfolio.
  - (b) Calculate the relative contribution of each stock to this portfolio's variance.
  - (c) Calculate the  $\beta$  of each stock relative to this two-stock portfolio. Check your results.
  - (d) Now calculate portfolio risk (standard deviation) and return using some different values for the weights and plot the results in risk-return space. Use the returns in Table 3.7 in the book (Cisco 7.5%, Amazon 12.5%)
  - (e) The graph you plotted under (d) gives you a good idea what the minimum variance portfolio looks like, but can you calculate its properties exactly? What weights give the portfolio its minimum variance? What are this portfolio's standard deviation and return?
  - (f) How would the graph you plotted under (d) look if Cisco and Amazon were perfectly positively correlated?
2. Recall your uncle Bob's portfolio problem in Tables 3.7 and 3.8. On the next birthday party, uncle Bob boasts that he has optimized his portfolio of 5 stocks and he demonstrates the calculation of the minimum variance portfolio using your spreadsheet. As Table 3.8 shows, the minimum variance portfolio has an expected return of 9.2% and a standard deviation of 23.2%. Your aunt Agatha is so impressed by the demonstration that she wants to invest her money in this minimum variance portfolio. You have collected some additional data about the risk free interest rate, which is 2.5%, and the expected return and risk of a broad index that represents the stock market as a whole. There are several index funds that follow this index very closely; they have an expected return of 12% and a standard deviation of 24%.
  - (a) Taking your additional information into account, is aunt Agatha's proposed portfolio mean-variance efficient?
  - (b) If not, how much extra return can she get with a portfolio that has the same standard deviation as the minimum variance portfolio in Table 3.8? What is the composition of that portfolio?
  - (c) Similarly, how much risk can she avoid with a portfolio that has the same return as the minimum variance portfolio in Table 3.8? What is the composition of that portfolio?
3. Stock A has a return of 10% and a standard deviation of 15%. Stock B has a return of 14% and a standard deviation of 20%. Their correlation coefficient is -1. Which combination of A and B gives the lowest variance? How high is that variance?

4. You have estimated regressions for a number of stocks and indices using 3 years of daily return data. For each stock you estimated the following regression equation:

$$r_{it} - r_{ft} = \alpha_i + \beta_i(r_{mt} - r_{ft}) + \varepsilon_{it}$$

where  $r_{it}$  is the return of stock  $i$  on day  $t$ ,  $r_{ft}$  is the risk free interest rate on day  $t$ ,  $r_{mt}$  is the return of the market index on day  $t$ ,  $\alpha_i$  and  $\beta_i$  are the regression coefficients obtained for stock  $i$ , and  $\varepsilon_{it}$  is the disturbance term. For a particular stock you estimated  $\alpha = 0.025$  with a standard error of  $\sigma_\alpha = 0.018$  and  $\beta = 1.46$  with a standard error of  $\sigma_\beta = 0.084$ .

- What does the CAPM predict for the values of  $\alpha$  and  $\beta$ ?
  - Are the estimation results in agreement with the prediction of the CAPM? Do an informal statistical test.
  - Is this stock riskier or less risky than the market as a whole?
5. The economy of the land Southway is dominated by two industries: oil production and fish farming. Hence, two pervasive economic risk factors determine the risk premia on the local stock market: the returns of the oil industry and the returns of the fish farming industry (which, in turn depend on the world market prices of oil and fish). There are three large mutual funds that hold well diversified portfolios of domestic stocks. You have made a summary of the funds' returns and their sensitivities for the two risk factors; Table 1 gives the summary.

Table 1: Funds' returns and sensitivities

Mutual fund	Exp. return	Sensitivities	
		Oil: $b_1$	Fish: $b_2$
1	0.172	1.2	0.8
2	0.208	0.9	1.4
3	0.136	0.5	0.7

- Describe how the sensitivities  $b_1$  and  $b_2$  can be obtained.
- Calculate the risk premia for oil price risk, fish price risk and the risk free interest rate.
- You want to divide your money over the three mutual funds in such a way that your portfolio has a sensitivity of 1 for the oil price risk and a sensitivity of 1 for the fish price risk. Can you make such a portfolio and, if so, how do you divide your money over three funds?